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QUANTIZATION BY COSMIC BACKGROUND RADIATION

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QUANTIZATION BY COSMIC BACKGROUND RADIATION

James Dehn

Abstract

In this paper we suggest that various modes in the cosmic background radiation field may account for the discrete properties exhibited by small systems. In particular, this view is applied to the one-, two- and three-dimensional oscillators and the hydrogen atom, systems which were treated by Schrodinger in his first papers on quantum mechanics. The usual energy formulas for the above systems are derived using this point of view, together with some indication of how transition probabilities might also be calculated. A connection between de Broglie's associated wave and a free mass moving in the cosmic background is also discussed. Analogs of the uncertainty and correspondence principles are briefly mentioned as are some of the implications this view might have for interpreting quantum theory. In this view particles and waves are separate, interacting entities and not complementary aspects of the same thing.

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1. Introduction

In 1987 we celebrated the centennial of the birth of Erwin Schrodinger who devised the wave-mechanical version of quantum mechanics and showed its equivalence to the matrix formulation of Heisenberg [1]. Schrodinger was inspired by de Broglie's idea that a wave should be associated with particle motion. He wrote a wave equation which showed how the integers of earlier theories could arise in a natural way. However, in spite of the success of his method, he was not satisfied with his own creation. At the end of the fourth part of his series of papers on Quantization as a Problem of Proper Values he discussed the physical significance of his wavefunction as the amplitude of a weight function in configuration space. In this matter he considered it a paradox that a system configuration should be a superposition of all imaginable configurations. He was uncomfortable with such a description and supposed that

> "there is something tangibly real behind the present conception also, namely, the very real electrodynamically effective fluctuations of the electric space density. The Ψ-function is to do no more and no less than permit the totality of these fluctuations being mastered and surveyed mathematically by a single partial differential equation." [2]

In other words, Schrodinger viewed quantum mechanics as a very useful tool which does not give a completely satisfactory description of reality, a minority opinion which he shared with Einstein. In particular, he viewed the notion of energy levels as a calculational convenience rather than a basic reality which permitted no further speculation. In his paper on The Exchange of Energy according to Wave Mechanics he expressed his distrust of the "axiomatic unintelligibility" of quantum postulates and his preference for some kind of resonance theory. In particular, if someone were to object that these postulates have been confirmed beyond doubt by experiment, he would reply:

> "Yes, I do question whether it is not very much more to the point to push the idea of the frequency of the de Broglie wave into the foreground....
>
> "I cannot help feeling that to admit the quantum postulates *in addition to* the resonance phenomenon is to accept *two* explanations for the same thing." [3]

2. Cosmic Background Radiation

Although a remnant cosmic background radiation was predicted as early as 1948 [4], it was not until 1965, four years after Schrodinger's death, that measurements by Penzias and Wilson [5] revealed the presence of an isotropic, unpolarized radiation field falling upon earth without seasonal variation. Dicke and co-workers [6] assumed that this radiation fills the universe and interpreted it to be the remnant of a "big bang" in which the universe had its origin. According to this model, very early in its history the universe was a relatively compact oven in which radiation coexisted with matter at temperatures in excess of 10^{10} K, presumably with a spectral peak near a frequency $\nu = 10^{21} s^{-1}$ (wavelength $\lambda = 10^{-11}$ cm). As the universe expanded, it cooled down until the background radiation field at present has a peak near $\nu = 10^{11} s^{-1}$ ($\lambda = 10^{-1}$ cm) corresponding to the background temperature T_B= 3.5K reported by Penzias and Wilson. This cooling was accompanied by the gradual appearance of ever larger and more complex stable structures. There has been much speculation (mostly cosmological) about the details of this evolutionary process. In such discussions it is usually assumed that the cosmic background field is only weakly coupled to matter at present. In this paper we will consider how the cosmic background field might be coupled to very small "quantum mechanical" bodies and exercise considerable influence on them, even though its influence might be negligible for the bodies we directly experience as well as for much larger "cosmological" bodies. In effect we are assuming that the cosmic background field is the something "tangibly real" Schrodinger was convinced lay behind the computational success of quantum mechanics. In particular, we will derive energy formulas for the simplest atomic and molecular systems discussed by Schrodinger in the collection of papers cited above. Along the way we will mention analogs of the uncertainty and correspondence principles and indicate how transition probabilities and intensity formulas might be calculated. In the conclusion we will briefly describe how this theory might fit into current discussions of the meaning of quantum theory.

Since quantum particles are now known to be residing in a bath of fossil radiation, it seems natural to assume that the particle-wave duality which they manifest may have its origin in this fact. In particular, the duality of vacuum fluctuations inducing particles to radiate while particles exhibit radiation reaction may be explained in this way. Dowling has summarized the literature on this subject and has pointed out that any separation of vacuum fluctuations and radiation reactions is purely mental [7]. Either (or both) points of view may be adopted. Such fluctuations/reactions are usually invoked to account for phenomena like natural line widths. Here we will consider the possibility that they might also explain the quantization of particle energy states.

The scalar field of an arbitrary charge distribution can be represented by a sum over multipole potentials. Similarly, the field of an arbitrary current distribution can be represented by a multipole expansion of the vector potential. Likewise, an electromagnetic radiation field can be represented as a mix of electric and magnetic multipole fields. Apart from a sinusoidal time factor, the general solution to Maxwell's equations may be written as [8]

$$\vec{B} = \sum_{\ell,m}[-(i/k)a_M(\ell,m)\nabla x g_\ell(kr)\vec{X}_{\ell m} + a_E(\ell,m)f_\ell(kr)\vec{X}_{\ell m}] \tag{1}$$

$$\vec{E} = \sum_{\ell,m}[\ (i/k)a_E(\ell,m)\nabla x f_\ell(kr)\vec{X}_{\ell m} + a_M(\ell,m)g_\ell(kr)\vec{X}_{\ell m}] \tag{2}$$

where the coefficients $a_E(\ell,m)$ and $a_M(\ell,m)$ specify the mix of electric and magnetic multipoles in the magnetic field, $\vec{B}$, and the electric field, $\vec{E}$. The radial functions $f_\ell(kr)$ and $g_\ell(kr)$ with $k=2\pi/\lambda=\omega/c$ are linear combinations of spherical Hankel functions of the first and second kinds. The vector spherical harmonics are

$$\vec{X}_{\ell m} = [1/\sqrt{\ell(\ell+1)}]\ \vec{L}\ Y_{\ell m}(\theta,\varphi) \tag{3}$$

where the angular momentum operator (transverse to the radial unit vector $\hat{r}$) is $\vec{L}=-i(\vec{r}x\nabla)$ and $Y_{\ell m}(\theta,\varphi)$ are scalar spherical harmonics. Here we desire solutions near the origin so we may take $f_\ell(kr)$ and $g_\ell(kr)$ to be the spherical Bessel functions $j_\ell(kr)$. We are also interested in the long wavelength approximation with $kr<<1$ so

$$j_\ell(kr) \approx (kr)^\ell/(2\ell+1)!! \tag{4}$$

and the radial factors in Eqs (1) and (2) decrease as ℓ increases for given r. Eq (3) can be written as a linear combination of scalar spherical harmonics with the same ℓ and m raised or lowered by unity or left unchanged. Consequently, the second terms in Eqs (1) and (2), namely, $\vec{B}^{(E)}_{\ell m}$ and $\vec{E}^{(M)}_{\ell m}$, both have the same magnitude determined by $j_\ell(kr)$. The first terms in these equations, namely, $\vec{B}^{(M)}_{\ell m}$ and $\vec{E}^{(E)}_{\ell m}$, are $(\pm i/k)j_\ell(kr)$ times the curl of Eq (3). Similarly, we may choose cylindrical or plane wave expansions [9].

3. Harmonic Oscillator

Here we will begin with a one-dimensional molecular oscillator which has no need of angular factors (ℓ=m=0). Since the infrared wavelengths of interest are much larger than the molecular dimensions, the cosmic background field may be represented by a Fourier series harmonic in time with constant coefficients:

$$F = \sum_{n=0}^{\infty} F_n = \sum_{n=0}^{\infty} f_n \exp[i(n+1)\omega t] \qquad (5)$$

A similar three-dimensional representation for periodic solutions in a large box leads to an equivalence between Maxwell's equations and the equations of motion of a set of harmonic oscillators [10]. We note that the frequency ω in Eq (5) is arbitrary and may be chosen to fit the needs of a particular problem. A Fourier integral representation may be more appropriate for some problems.

In calculating the energy density of this field the frequency of occurence of the n^{th} normal mode with energy $nh\nu$ is $\exp[(-nh\nu)/(kT)]$ where h is Planck's constant and k is Boltzmann's constant [11]. If we let $a = (h\nu)/(kT)$, the average energy is

$$\frac{\sum_{n=0}^{\infty}(nh\nu)e^{-na}}{\sum_{n=0}^{\infty} e^{-na}} = \frac{h\nu\ e^{-a}/\ (1-e^{-a})^2}{1\ /\ (1-e^{-a})} = \frac{h\nu}{(e^{a} - 1)} \qquad (6)$$

Multiplication of this equation by the number of modes per unit volume in a frequency interval $[(8\pi\nu^2)/c^3]$ where c is the speed of light gives Planck's law as is well known.

When Schrodinger [12] applied his theory to the harmonic oscillator with reduced mass m_o, he introduced the displacement amplitude $A=\sqrt{\hbar/(m_o\omega)}$, where $\hbar=h/(2\pi)$ and $\omega=2\pi\nu$. Classically this is multiplied by an harmonic time factor like $\cos(\omega t)$, so the second time derivative or force per unit mass is proportional to $\omega^2 A=\omega\sqrt{(\hbar\omega)/m_o}$. The amplitude of the force per unit mass, f_n, exerted by the n^{th} mode of the cosmic background should be proportional to this factor as well as to the frequency of occurence of this amplitude, which we take to be the square root of Boltzmann's factor, $\exp(-na)$. We also (somewhat arbitrarily) take the proportionality constant to be $\exp(-a/2)$ which insures that this force is very small for frequencies of interest. For example, if $\nu>10^{13}$ then $a>137$ for $T_B\approx 3K$ at present. If $m_o\approx 10^{-24}$ g for a small molecule, then $m_o f_o \approx 10^{-35}$ dyne. Thus we assume that

$$f_n = \omega\sqrt{\hbar\omega/m_o}\ \exp[-(n+1)(a/2)] \qquad . \qquad (7)$$

The field in Eq (5) has no time-independent component and represents an infinite collection of indistinguishable photons. The field intensity of a component is found as usual by squaring the amplitude, using the complex conjugate so $F_n F_n^*$ is independent of time. We may use $F_n F_n^*$ instead of Boltzmann's factor in Eq (6) to find

$$\sum_{n=0}^{\infty}(nh\nu)(F_n F_n^*) \;\; / \sum_{n=0}^{\infty}(F_n F_n^*) = h\nu/(e^a - 1) \tag{8}$$

which again gives Planck's law when multiplied by an appropriate factor, since the constant parts of $F_n F_n^*$ cancel in numerator and denominator. Similar results can be obtained from spherical or cylindrical wave representations by integrating over orthonormal radial and angular functions.

In the macroscopic world an undamped oscillator is known to be an unrealistic idealization and to maintain the oscillations, forcing must be included as well. Let us assume that this is also true for molecular oscillators and include a damping term in the equation of motion. A linear damping constant has the dimensions of a frequency and to be commensurate with the force above should have the form

$$b = \delta\omega \exp(-a/2) \tag{9}$$

where δ is a dimensionless constant. The equation of motion is then

$$x + b\dot{x} + \omega^2 x = \delta F \tag{10}$$

as for any linearly damped, forced harmonic oscillator. Here x is the displacement from equilibrium while ω is the natural frequency of the oscillator. We could use ω_o instead and then choose the arbitrary frequency $\omega = \omega_o$ in Eq (5), but this amounts to the same thing. The dimensionless factor δ is for example,

$$\delta = \mu/(er_o) \ll 1 \tag{11}$$

where μ is the electric dipole moment (possibly induced) of a diatomic molecule, $r_o \gg x$ is the equilibrium interatomic distance, and e is the electronic charge. If $\omega \approx 10^{14}\ s^{-1}$ so $a > 137$ as mentioned above, then $b < 10^{-16}\ s^{-1}$. The solution to the homogeneous equation is a transient proportional to $\exp[-(b/2)t]$ and would not yet be negligible if b were this small for 10^{17} s $\approx 10^{10}$ years, using present estimates of the age of the universe. Of course in a younger, hotter universe Eq (10) probably did not apply. Most likely a nonlinear description would be needed. For sudden changes we usually expect large transients, but for a very gradual evolutionary cooling during the time when Eq (10) might apply we expect negligible transient amplitudes to begin with. We will assume them to be zero and retain only the solution to the inhomogeneous equation. This has the usual form

$$x = \sum_{n=0}^{\infty} A_n \exp\{i[(n+1)\omega t - \alpha_n]\} \tag{12}$$

with

$$A_n = \delta f_n / \sqrt{[(n+1)^2\omega^2 - \omega^2]^2 + b^2(n+1)^2\omega^2} \tag{13}$$

and

$$\tan \alpha_n = b(n+1)\omega / [\omega^2 - (n+1)^2\omega^2] \quad . \tag{14}$$

For $n>0$, $\alpha_n \approx 0$ since $b \ll \omega$, while $\alpha_0 = \pi/2$. For $n>0$ we can neglect $b^2(n+1)^2\omega^2$ in Eq (13), so

$$A_n = (\delta f_n/\omega^2) / (n^2+2n) < (\delta f_n/\omega^2)/n^2 \tag{15}$$

while for n=0,

$$A_0 = \delta f_0/(b\omega) = \sqrt{\hbar/(m_0\omega)} \tag{16}$$

from Eqs (7), (9) and (13). Clearly the n=0 term dominates in Eq (12). In fact, it can be larger than all the other terms put together. From Eqs (7) and (15) we find

$$\sum_{n=1}^{\infty} A_n < (\delta/\omega^2) \sum_n (f_n/n^2) < \sqrt{\hbar/(m_0\omega)}\, e^{-a/2} [\sum_n (e^{-na/2}/n^2)] \quad . \tag{17}$$

The summation in the last form of Eq (17) is the dilogarithm and is equal to Spence's integral with a maximum value near unity [13]. In this case, if we use Eq (16) in Eq (17), we find $A_0 > \sum_n A_n$ as stated above. From Eq (15) we note that A_n decreases with increasing n in such a way that $A_n > \exp(a/2)A_{n+1}$.

For the cosmic background field we have used the traditional equivalence to an infinite set of oscillators with discrete frequencies. Since the background is a continuum with all frequencies present, let us consider perturbations by frequencies which are arbitrarily close to the natural frequency ω. If we let $q=(n+1)\omega\approx\omega$ for $n\approx0$ instead of integer, we see that f_n in Eq (7) is practically independent of q under these conditions, while b in Eq (9) is exactly independent of q. From Eq (13) we see that $A^2(q)=.5A^2(\omega)$ for $(q-\omega)\approx.5b<10^{-16}$, $q\approx\omega$ and $q+\omega\approx2\omega$ so the intensity is reduced to half its peak value for very small departures from ω. The smallness of b makes the peak very sharp at present, and as the universe continues to cool, $T_B\to0$ and $b\to0$, so the peak approaches a delta function.

Because of the dominance of A_o, Eq (12) is approximately

$$x \approx A_o \exp[i(\omega t-\pi/2)] = (-i)A_o\exp(i\omega t) \tag{18}$$

and the $x,\dot{x}$ phase trajectory given by Eq (12) is a slightly perturbed ellipse, which would be an asymptotic limit cycle if there were a transient. If we multiply Eq (10) by $m_o(\dot{x}dt)^*=m_o(dx)^*$ and integrate to find the oscillator energy we see that

$$E \approx .5m_o(\omega A_o)^2 = .5\hbar\omega = E_o \tag{19}$$

when we use Eq (16). This is the expectation value of the energy over times long compared to a period $T=2\pi/\omega$ since the sum of all the other terms arising from δF and $b\dot{x}$ are negligible by comparison. Integrals over products with unequal n vanish exactly, while the sum over terms with equal n vanishes approximately over the observation times to which we are limited in spectroscopy.

We note that A_o in Eq (16) is about 10^{-9} cm if $m_o=10^{-24}$ g and $\nu=10^{14}$ s^{-1}, a reasonable estimate for a vibration amplitude. For much larger masses like those of our direct experience, A_o is completely negligible unless the frequency is extremely small, requiring observation times beyond our abilities. This agrees with the idea that the cosmic background radiation field has a negligible effect on large masses and is the analog of the correspondence principle. However, this does not rule out the possibility that this radiation field might influence the motion of a slowly oscillating universe. Of course such a speculation cannot be affirmed or denied because our longest observation times are so severely limited compared to what would be required.

Eq (19) is not quite exact since the lead term in Eq (12) is perturbed by other modes in the background. Without these perturbations, the root mean square amplitude over a period would be $A_o/\sqrt{2}$, since the average of the sine squared or cosine squared is 1/2, using the real part. Similarly, the rms momentum would be $m_o\omega A_o/\sqrt{2}$ and the product of these two quantities would be $m_o\omega A_o^2/2 = \hbar/2$. However, because of the perturbations, the actual amplitude, A, is slightly larger than A_o. Consequently,

$$m_o\omega A^2/2 > \hbar/2 \tag{20}$$

which is the analog of Heisenberg's uncertainty principle applied to the harmonic oscillator. In Schrodinger's wave mechanics we arrive at this result by using the ground state wave function, a Gaussian distribution in configuration space. Here we are using the rms amplitude of a perturbed oscillator in physical space. Of course we cannot measure this amplitude any more than we can measure the Gaussian distribution. The best we can do is measure frequencies which are absorbed or emitted over many vibration periods.

Schrodinger considered it a paradox that quantum mechanics should describe a system as a superposition of all imaginable configurations and supposed that there must be some "tangibly real" frequencies responsible for spectroscopic observations. In the present view we see that the real cosmic background frequencies lead to a superposition of all possible states in Eq (12). With only the cosmic background present (in the absence of an observer) the oscillator exists with one state dominant. If an observer adds a second electromagnetic field containing one or more of the higher frequencies in Eq (12) in equal amounts, then resonance can occur with one or more of the modes already present. If the second field is strong enough, an observation over many periods tells us that some of the applied energy has been absorbed. This leads us to say that transitions have occurred to higher energy states. In the present theory the dominant term in Eq (12) for a particular molecule can become one of the higher modes. In an ensemble of identical molecules individual molecules will be vibrating in different modes, shifting from one mode to another as photons are absorbed and emitted. At any moment most will vibrate in the lowest mode with successively fewer in higher modes. The relative number in each mode will be governed by the likelihood of occurence of each mode in the background field if all modes are equally represented in the applied field. If a particular mode dominates in the applied field, this field can govern the distribution and population inversions are possible.

The applied field can also be represented by a Fourier series with coefficients controlled to a great extent by an experimenter in the laboratory. The interaction energy is the expectation value

$$\epsilon = -\mu \int_o^T E_a(\dot{x}dt)^* \tag{21}$$

where E_a now stands for the external field represented by a Fourier series. The solution for x in Eq (12) will have added to it terms with amplitudes similar to Eq (13) but with the coefficients of the E_a series replacing the f_n. These new amplitudes can add to the old ones and determine new dominant terms as explained above. Only terms with matching frequencies do not vanish in the integration over a period in Eq (21). In this theory we note that transitions to any frequency mode which is reinforced can occur even in the linear approximation. The assumption that f_n is a probability amplitude has already been used in deriving Planck's law above and can be used again here to account for the reduced intensity of higher frequency absorption lines. As we know, Schrodinger disliked the interpretation of his wave function as a probability amplitude. Perhaps he would have liked the present description better since he was a disciple of Boltzmann.

From what has been said so far, it is clear how this theory can describe the fact that integer multiples of a fundamental frequency are absorbed by an ensemble of molecules in an applied field. This can also be summarized by using the traditional energy level scheme

$$E_n = (n + .5)\hbar\omega \tag{22}$$

with n=0 giving Eq (19), agreeing with Schrodinger's result [12]. As usual, the fact that the higher frequencies absorbed are not quite evenly spaced on a frequency scale can be accounted for by using an anharmonic restoring force. A nonlinear restoring force of the "softening" type in Eq (10) can account for the fact that the line spacing decreases slightly as the frequency increases. This is also required to explain dissociation.

In summary, we are assuming a continual interaction between radiation and matter which is ususally elastic, that is, without detectable energy change in either field or particle. Occasionally an inelastic interaction will occur and a matter particle will acquire a new mode of vibration from the field, even in the almost empty space between galaxies. Since cosmic background photons are ordinary photons their action is enhanced by stellar photons. The greater photon density inside galaxies leads to a greater frequency of excited particle states, so we can observe the spectra of interstellar molecules like CH and CN. Further enhancement occurs on the surface of a planet like the earth which is close to a sun, and even greater enhancement occurs inside a star like our sun. In addition to such natural variations in photon density, man has learned to produce local variations in the intensity and composition of electromagnetic fields such that population inversions are even possible. Such added fields mask but do not eliminate the effects of the cosmic background field.

4. Free Mass

If there is no restoring force in Eq (10) then $\omega^2=0$ and this equation can describe a small mass like an atom or electron moving with constant velocity ($\ddot{x}=0$) in the isotropic background field. Since the distances covered can be much larger than a wavelength we can no longer use the long-wavelength approximation. Let r be the particle coordinate to emphasize the fact that we are no longer dealing with a small vibration amplitude, x. Let R be the location of a point on a travelling wave in the background field. Eq (10) with $\delta=1$ becomes

$$b\dot{r} = bv = f_o \exp[i(\omega t - kR)] \qquad (23)$$

for a wave moving in the same direction with n=0. Both v and the phase are constants so $\dot{R}=c=\omega/k$. Squaring Eq(23) in the usual way gives $v^2=(f_o/b)^2=h\nu/m_o$ by Eq (16), a relation which may also be written as

$$h/(m_o v) = v/\nu = \lambda \qquad (24)$$

which defines a wavelength λ associated with a particle of momentum $m_o v$ moving in the cosmic background. In deriving Eq (24), de Broglie [14] defined the wavelength to be V/ν where $V=c^2/v$ is a superluminal phase velocity and $\nu=W/h$ for relativistic energy $W=m_o c^2/\sqrt{1-(v/c)^2}$. When $(v/c)^2<<1$, $\nu=m_o c^2/h$ and $\lambda=(c^2/v)/(m_o c^2/h)$ which is Eq (24). Here we have derived the same relation by starting with the motion of a non-relativistic particle moving in the cosmic background field. This gives a physical basis for Eq (24) and eliminates the need to postulate an associated wave.

Before continuing let us make some qualitative remarks. The cosmic background field (often together with other photon fields) continually interacts with every particle in the world which possesses a permanent or induced electromagnetic multipole moment of any order. For example, if free particles of this type pass through a slit (or double slit or crystal) one at a time and register on a detector screen, each free particle as well as the particles composing the slit walls and screen interact with the background (and other) fields, modifying them and being modified by them during the entire time of an experiment. In this way they continually interact with each other, communicating at the speed of light. If the slit width is sufficiently small compared to the particle wavelength in Eq (24), diffraction effects can be detected. Particles passing through the center of a slit are undeflected since they are equally influenced by each wall. Other particles will be deflected if they pass closer to one wall than the other, because a greater pressure is exerted by the electromagnetic field emanating from the nearer wall. Analogies might be made with the radiation pressure exerted on the particles of a comet passing near the sun. The comet develops a large tail pointing away from the sun and a small tail (sometimes visible) pointing toward the sun. The radiation pressure exerted by the modified cosmic background and other electromagnetic fields emanating from slit walls is very weak by comparison, but the particles deflected are comparably small. We cannot control the exact approach path of each particle and so control its deflection. However, we can describe the overall deflection pattern in terms of wave theory, using the wavelength and phase as well as the slit widths and separations as parameters. Each particle passes through a particular part of a particular slit. However, its associated cosmic background wave passes through all parts of each slit. Thus electromagnetic waves replace Schrodinger's waves in ordinary theory. Similarly, they replace the pilot waves of de Broglie-Bohm theory or the background fluctuations of Dirac theory.

If the walls in a single slit experiment are geometrically identical and are made of the same material, the resulting diffraction pattern will be symmetric about a center line parallel to the slit. If one wall is removed the pattern is asymmetric. Most of it will lie on the open side of the straight edge, but a few particles will be bent behind the wall. This is analogous to the large and small tails on a comet near the sun. If both walls are geometrically identical but each is made of a different material, say beryllium and uranium, the pattern may also be asymmetric with respect to the previous center line. This could indicate that different materials modify the background fields differently, resulting in a net lateral component of deflection even at the center of the slit. We might also expect geometrically identical circular apertures in walls made of different materials to produce different patterns of concentric circles, if such modification differences are strong enough.

Neutrons have no electric monopole (charge) and may or may not have a very small electric dipole moment. Their diffraction is due principally to their magnetic dipole moment. What about fields other than electromagnetic? There might be as yet undetected cosmic background fields corresponding to nuclear, weak and gravitational forces. However, we shall not speculate about them here.

A quantitative treatment of these ideas is desirable and will be attempted in another place. Now let us return to the program we have outlined for this paper.

5. Rigid Rotator

Since $r=r_o$ is constant for the rigid rotator and $\nabla^2 Y_{\ell m}=[\ell(\ell+1)]/r_o^2]Y_{\ell m}$, the curl reduces to [15]

$$\nabla x \vec{X}_{\ell m} = -i(\hat{r}/r_o)\sqrt{\ell(\ell+1)}\, Y_{\ell m}(\theta,\varphi) \quad . \tag{25}$$

Consequently, the first terms in Eqs (1) and (2) have the same magnitude which is proportional to $[j_\ell(kr_o)/(kr_o)]$. From Eq (4) and $kr_o<<1$ we see that the second terms in Eqs (1) and (2) are negligible compared to the first for given ℓ.

The amplitude in Eq (7) must be multiplied by the above radial and angular factors for problems with spherical symmetry like the rigid rotator and hydrogenlike atoms. When we do this the amplitude is not isotropic. However, the expectation value of the intensity obtained by squaring and integrating over the volume will be isotropic. Here we are interested in the near-field, long-wavelength approximation subject to the constraint $r=r_o$ so we can represent the cosmic background field modes by

$$F_n = f_n \exp[i(n+1)\omega t] \sum_{\ell,m} [j_\ell(kr_o)/(kr_o)] \sqrt{\ell(\ell+1)}\, Y_{\ell m}(\theta,\varphi) \quad . \tag{26}$$

The equations of motion for the three-dimensional isotropic oscillator are then (letting $\delta=1$ for simplicity)

$$\ddot{x}_j + b\dot{x}_j + \omega^2 x_j = \sum_n F_n = F \tag{27}$$

with $j=1,2,3$. For variable r, the F_n which represent the electric or magnetic field components will have more general forms than Eq (26). However, for the rigid rotator Eq (26) applies since we have the constraint

$$\sum_j x_j^2 = r_o^2 \quad . \tag{28}$$

The first time derivative of Eq (28) gives

$$\sum_j x_j \dot{x}_j = 0 \quad , \tag{29}$$

while the second time derivative gives

$$v^2 = \sum_j \dot{x}_j^2 = -\sum_j x_j \ddot{x}_j = \omega^2 r_o^2 - F \sum_j x_j \quad . \tag{30}$$

The last form in Eq (30) was obtained by using Eqs (27) to (29). The expectation value of Eq (30) is $<v^2>=\omega^2 r_o^2$. The solutions to the undamped, unforced form of Eqs (27) are sinusoidal functions of time with different amplitudes and phases in the absence of the constraint. If the phases are the same we have pure vibration. If the amplitudes are the same and the phases are properly chosen, we have pure rotation. When we add the damping and forcing perturbations given above, we must keep in mind the fact that in the spherical harmonics $Y_{\ell m}(\theta,\varphi)$ θ and φ are not functions of time but express the radiation patterns experienced by the rotator. All time dependence for the solutions of Maxwell's equations is expressed in the exponential factors. Consequently, a differentiation of a solution, x_j, with respect to time leads to multiplication by ω.

If we let the amplitude $A_o=r_o$ in Eq (16) and solve for ω we find

$$\omega = \hbar/(m_o r_o^2) = \hbar/I \tag{31}$$

when we introduce the moment of inertia of the rotator $I=m_o r_o^2$. The components of the angular momentum involve sums of products like $m_o x_i \dot{x}_j$ which are proportional to $m_o r_o^2 \omega = I\omega = \hbar$ by Eq (31). A generalization of Eq (13) leads to

$$A_{n\ell m} = A_n[j_\ell(kr_o)/(kr_o)] \sqrt{\ell(\ell+1)}\; Y_{\ell m}(\theta,\varphi) \tag{32}$$

when we use Eq (26). The expectation value of the amplitude squared is found by integrating over the volume with suitable normalization. For example, we find $<A^2_{o\ell m}>= r_o^2 \ell(\ell+1)$. A time differentiation leads to multiplication by ω and to $<v^2>=\omega^2 r_o^2 \ell(\ell+1)$, so the rotational energy is

$$E_r = .5m_o<v^2> = .5I(\hbar/I)^2 \ell(\ell+1) = \ell(\ell+1)\hbar^2/(2I) = L^2/(2I) \tag{33}$$

when we use Eq (31). This agrees with Schrodinger's result [16]. Here L^2 is the square of the total angular momentum. The dipole term with $\ell=1$ gives the lowest rotational energy, while $\ell=0$ would correspond to pure vibration.

If, for example, the rotator is a heteronuclear diatomic molecule with a dipole moment, it can interact with a second electromagnetic field added to the cosmic background. This second field may also be represented as a sum over frequency modes and multipoles. Of course in a laboratory the expansion coefficients in the representation of the applied field can be controlled to a considerable extent, unlike the cosmic background. When resonant frequencies and multipoles are strong enough in the applied field, certain terms in the solutions will become dominant and the system will be observed to be in a new state. Since both fields depend on spherical harmonics, selection rules may be derived in the usual way. The addition of stationary electric or magnetic fields will destroy the spherical symmetry and require the use of m values higher than unity [17]. Of course a rigid rotator is an idealization for a molecule. The vibrating rotator can be treated as usual by using perturbation methods.

Schrodinger also considered the case in which the axis of rotation is fixed, that is, a two-dimensional isotropic oscillator subject to the constraint $r=\sqrt{x^2+y^2}=r_o$. In this case we can use cylindrical functions to represent electromagnetic fields, including the cosmic background field [9]. Instead of Eq (26) we have

$$F_n = f_n \exp[i(n+1)\omega t] \sum_k k\, J_k(\lambda r_o) \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (k\varphi) \tag{34}$$

for the dominant terms in the near-field, long-wavelength approximation. Here λ is the propagation constant. Since k appears in Eq (34) instead of $\sqrt{\ell(\ell+1)}$ as in Eq (26), it is clear that the rotational energy for a fixed axis is

$$E_r = .5 m_o \langle v^2 \rangle = k^2 \hbar^2/(2I) = L^2/(2I) \tag{35}$$

which agrees with Schrodinger's result [18]. In this case, since $\langle v^2 \rangle = r_o^2 \omega^2$, the angular momentum is

$$L = m_o r_o^2 \omega = I\omega = k\hbar \tag{36}$$

from Eq (35). From Eq (36) with k=1,2... we see that the frequency of a two-dimensional oscillator subject to a circular constraint is quantized, a result we will use in the next section.

6. Hydrogen Atom

Connections between Schrodinger's equations for the hydrogen atom, the spherical rotator and the four-dimensional isotropic oscillator (expressed in terms of generalized Euler angles) were discussed by Ikeda and Miyachi in 1970 [19]. Five years earlier, Kustaanheimo and Stiefel had generalized the Levi-Civita tranformation and showed the equivalence of the three-dimensional Kepler problem of celestial mechanics to the four-dimensional isotropic oscillator [20]. Ikeda and Miyachi did not use their results. However, in the last fifteen years many workers have realized the usefulness of the Kustaanheimo-Stiefel (K-S) transformation in discussing the quantum mechanics of the hydrogen atom, especially in stationary electric or magnetic fields, and have extended the discussion to all formulations of quantum mechanics. The literature on this subject has recently been summarized by Kibler and co-workers [21]. Even more recently, Chen [22], has used a variation of the K-S transformation to show that the classical Coulomb-Kepler problem in three dimensions is equivalent to a pair of classical two dimensional oscillators possessing the same angular momentum, a result he had previously derived in the quantum mechanical case [23]. Chen notes that an application of the Sommerfeld-Wilson quantization rules to the classical case leads to the same results as in quantum mechanics. Chen and Kibler [24] have discussed the role of the K-S constraint condition in determining the phase relationship for the pair of two-dimensional oscillators. They point out that the equality of the angular momenta for the pair of oscillators also appears if squared parabolic coordinates are used to separate the Schrodinger equation for the hydrogen atom.

The solution of the classical Coulomb-Kepler problem is an elliptical orbit with angular momentum constant in magnitude and direction, as is well-known. Imposing an isotropic cosmic background field on this orbit will not change it, at least on average, since the same perturbation will be felt no matter what the direction of the angular momentum. Consequently, the orbit will on average remain an ellipse oriented in the same direction. The dominant part of the perturbed solution will be the unperturbed ellipse, just as the dominant part of the linear oscillator solution is the unperturbed solution.

There are many ways to approach the perturbed hydrogen atom problem. Since the perturbations are small and periodic in time, we might follow the advice of Kustaanheimo and Stiefel [25] and use the two-dimensional Levi-Civita transformation with regularization of the time. This suggestion has been developed for the Kepler problem by Stiefel and Scheifele [26]. On the other hand, we might use Chen's transformation reduced to two dimensions without regularization. One of Chen's Euler angles is $\alpha=\theta/2$ where θ is the polar angle in spherical coordinates. If $\theta=\pi/2$ so the motion occurs in the x,y plane then Chen's circular oscillator radii u and v are equal. Since the angular momenta are equal, the angular frequencies must also be equal and the angles differ by an arbitrary constant which we may choose to be zero. Still another approach might be to regularize the motion without any transformation of the dependent variables. This possibility was mentioned in passing by Stiefel and Scheifele [27] but not developed at all. Here we will pursue this last approach, since it is worth developing and leads rather directly to the energy formula we wish to derive.

The vector equation of motion is

$$\ddot{\vec{r}} + (K^2/r^3)\vec{r} = \vec{F} - b\dot{\vec{r}} \tag{37}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in cartesian coordinates and $K^2 = Ze^2/m_o$ for a hydrogenlike atom with atomic number Z, reduced mass m_o, and electronic or protonic charge e. If we number the coordinates in the usual way the components of Eq (37) are

$$\ddot{x}_j + (K^2/r^3)x_j = F_j - b\dot{x}_j \tag{38}$$

where modal expansions can be used for the components of the isotropic field, F. Here

$$r^2 = \sum_j x_j^2 \tag{39}$$

with j=1,2,3. Time differentiation of Eq (39) gives

$$r\dot{r} = \sum_j x_j\dot{x}_j \tag{40}$$

while time differentiation of Eq (40) gives

$$r\ddot{r} + \dot{r}^2 = \sum_j (\dot{x}_j^2 + x_j\ddot{x}_j) \quad . \tag{41}$$

We can take the scalar product of $\vec{r}$ with Eq (37) or multiply each Eq (38) by the corresponding $\dot{x}_j$ and add the results to find

$$\sum_j \dot{x}_j\ddot{x}_j + (K^2/r^3)(r\dot{r}) = \sum_j (F_j\dot{x}_j - b\dot{x}_j^2) \tag{42}$$

where we have used Eq (40). The value of the right side of Eq (42) over times long compared to a period is expected to be zero, so the integral of the left side gives the expectation value of the energy per unit mass

$$E/m_o = .5 \sum_j \dot{x}_j^2 - K^2/r \tag{43}$$

as usual. Similarly, we can take the vector product of $\vec{r}$ with Eq (37) and integrate to find that the angular momentum components have constant expectation values. If we put Eq (43) in Eq (41), we obtain

$$\sum_j x_j \ddot{x}_j = (r\ddot{r} + \dot{r}^2) - 2(E/m_o + K^2/r) \quad . \tag{44}$$

Now take the scalar product of $\vec{r}$ with Eq (37) to find

$$\sum_j x_j \ddot{x}_j + (K^2/r^3)(\sum_j x_j^2) = \sum_j (F_j x_j - b x_j \dot{x}_j) \tag{45}$$

and use Eqs (39), (40) and (44) in Eq (45) multiplied by r to obtain

$$r(r\ddot{r} + \dot{r}^2) - (2E/m_o)r - K^2 = r(\sum_j F_j x_j - br\dot{r}) \quad . \tag{46}$$

Finally, let us introduce the fictitious time τ defined by the relation $dt = r d\tau$ to regularize Eq (46). This gives $r' = r\dot{r}$ and $r'' = r^2\ddot{r} + (r')^2/r = r(r\ddot{r} + \dot{r}^2)$ where a prime denotes differentiation with respect to τ. Eq (46) becomes

$$r'' + (2\omega)^2 r = K^2 + r(\sum_j F_j x_j - br') \tag{47}$$

where $(2\omega)^2 = -2E/m_o = 2h_K > 0$ since $E<0$ for an ellipse. This defines the positive energy per unit mass, h_K. The expectation value of the right side of Eq (47) is K^2, so the dominant part of the solution is

$$r = a - a\epsilon_o \cos(2\omega\tau) \tag{48}$$

where $a = K^2/(2\omega)^2 = K^2/(2h_K)$, or

$$-E/m_o = h_K = K^2/(2a) = 2\omega^2 \tag{49}$$

the usual result that the energy is independent of the eccentricity, ϵ_o, and depends only on the semi-major axis of the ellipse, namely, a. Eq (48) is the three-dimensional version of Eq (13), p. 39 of reference 26 and becomes the same if we let z=0, for example. Eq (49) is the same as Eq (18) in that reference. Stiefel and Scheifele obtained these results by using the Levi-Civita transformation with time regularization to show the equivalence of the Kepler problem and the motion of a two-dimensional oscillator. In terms of the fictitious time they find

$$x = a[\cos(2\omega\tau) - \epsilon_o] \quad \text{and} \quad y = a\sqrt{1-\epsilon_o^2}\,\sin(2\omega\tau) \quad . \tag{50}$$

It is interesting to note that the virial theorem also leads to eq (49). Since the average of the kinetic energy over a period is (-.5) times the average potential energy, the right side of Eq (43) becomes $[-K^2/(2r)]$ or Eq (49), since a is the average value of r, namely, half the sum of the aphelion and perihelion distances.

We can also use Eq (48) to find the time

$$t = \int r\, d\tau = a\tau - [(a\epsilon_o)/(2\omega)]\sin(2\omega\tau) \quad . \tag{51}$$

When t = T, a period, $\tau = \pi/\omega$, so Eq (51) becomes Kepler's third law:

$$T = a\pi/\omega = 2\pi\, a^{3/2}/K \tag{52}$$

since $\omega = K/(2\sqrt{a})$ from Eq (49). Eq (52) is also Eq (27) on p. 41 of reference 26. If we set the last form of Eq (52) equal to $2\pi/\omega$ and square, we find

$$a^3 = K^2/\omega^2 = (Ze^2/m_o)/\omega^2 \quad . \tag{53}$$

Since the expectation value of the energy in Eq (49) depends only on a and not on ϵ_o, it is the same for ϵ_o=0, $r=a=r_o$ in Eq (48) (a circle) as for any other value of ϵ_o which is consistent with an ellipse. Since Coulomb-Kepler motion is equivalent to a two-dimensional oscillator, we may consider the particular case of a circular oscillator or rigid rotator and let r_o=a in Eq (36) to find $\omega=(n\hbar)/(m_o a^2)$, using n=1,2... instead of k. Putting this into Eq (53) gives

$$a = n^2\hbar^2/(m_o Ze^2) \quad . \tag{54}$$

Putting Eq (54) into Eq (49) gives

$$E = -(m_o Z^2 e^4)/(2n^2 \hbar^2) \tag{55}$$

in agreement with Schrodinger's result when Z=1 [28].

7. Summary

In this paper we have suggested that the cosmic background radiation field continually exchanges energy with small bodies like atoms and molecules, providing them with a variety of modes through which they can interact with other systems. In this view particles and waves are separate entities which interact and are not complementary aspects of the same thing. We have sketched this theory only for the simple systems which were discussed by Schrodinger in his early papers on quantum mechanics and have been content for the most part with deriving his energy expressions. Clearly much remains to be done to complete this theory and extend it to other systems.

Let us reflect briefly on what Schrodinger might have thought of this description. He probably would have liked it, since it uses real frequencies to explain the success of his wave function. His wavefunction is viewed as one of the elegant ways which have been devised for surveying possibilities and probabilities rather than as an entity which "collapses" when an observation is made. This agrees with Schrodinger's own perception of his wavefunction and allows his cat to "walk by himself" as cats are accustomed to do. However, this description does not attempt to substitute waves for particles as Schrodinger did briefly. It does include probabilities, but after the manner of Boltzmann and probably to Schrodinger's liking.

Many of the measurement problems discussed in connection with quantum mechanics [29] do not arise in the present description, since only electromagnetic waves in physical space rather than Schrodinger waves in configuration space are used. In the context of such discussions, the present theory can be characterized as a realistic description of a single universe. It is holistic in the sense that the particles which compose the measuring apparatus as well as the system being measured are in constant communication via exchanges through (at least) the cosmic background field. In the same sense it is local if such exchanges are limited to the speed of light (unlike de Broglie's superluminal waves). Of course these comments by no means answer all of the questions which have been raised in such discussions and much remains to be done.

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